[2][a] $-1 < \cos n < 1$

$$1 < 2+\cos n < 3$$

$$\underbrace{0 <}_{\frac{1}{2}} \frac{2^{n-5}}{(3n)!} < \underbrace{\frac{(2+\cos n)2^{n-5}}{(3n)!} < 3 \cdot \frac{2^{n-5}}{(3n)!}}_{\frac{1}{2}}$$

CONSIDER $\sum_{n=1}^{\infty} \frac{2^{n-5}}{(3n)!}$

$$\underbrace{\lim_{n\to\infty} \left| \frac{2^{(n+1)-5}}{(3(n+1))!} \cdot \frac{(3n)!}{2^{n-5}} \right|}_{\frac{1}{2}}$$

$$= \lim_{n\to\infty} \left| \frac{2^{n-4}}{2^{n-5}} \cdot \frac{(3n)!}{(3n+3)!} \right|$$

$$= \lim_{n\to\infty} \left| 2 \cdot \frac{\cancel{(3n)!}}{(3n+3)(3n+2)(3n+1)\cancel{(3n)!}} \right|$$

$$(1)\ \underbrace{= \lim_{n\to\infty} \frac{2}{(3n+3)(3n+2)(3n+1)}}_{} \ \underbrace{= 0}_{\frac{1}{2}} \ \underbrace{< 1}_{\frac{1}{2}}$$

$$\text{SO } \frac{1}{2}\ \underbrace{3\sum_{n=1}^{\infty} \frac{2^{n-5}}{(3n)!} \text{ CONV (RATIO TEST)}}_{}$$

MUST HAVE "3" AS PART OF SUM

$$\text{SO } \sum_{n=1}^{\infty} \frac{(2+\cos n)2^{n-5}}{(3n)!} \ \underbrace{\text{CONV (COMP TEST)}}_{\frac{1}{2}}$$

[2][b] $\frac{1}{2}$ $0 < \left|\frac{\sin n^2}{n(1+\ln n)^2}\right| < \frac{1}{n(1+\ln n)^2}$ $\frac{1}{2}$ ← MUST HAVE ABSOLUTE VALUE

SEE NOTE AT END OF SOLUTION

$$\frac{d}{dx}\frac{1}{x(1+\ln x)^2} = -\frac{1(1+\ln x)^2 + x\cdot 2(1+\ln x)\cdot\frac{1}{x}}{(x(1+\ln x)^2)^2} \quad 1$$

$$= -\frac{(1+\ln x)[1+\ln x+2]}{x^2(1+\ln x)^4}$$

$$= -\frac{3+\ln x}{x^2(1+\ln x)^3} \quad \tfrac{1}{2} \qquad \ln x > 0 \text{ ON } [1,\infty)$$

< 0 $\frac{1}{2}$ $\longrightarrow$ $\frac{1}{x(1+\ln x)^2}$ DECREASING $\frac{1}{2}$

$\frac{1}{x(1+\ln x)^2} > 0$ $\frac{1}{2}$

$\frac{1}{x(1+\ln x)^2}$ CONT ON $[1,\infty)$ $\frac{1}{2}$

(DISCONTINUITIES ONLY AT $X=0$ AND $1+\ln x = 0$ IE, $x=\frac{1}{e}$
$0, \frac{1}{e} \notin [1,\infty)$)

$$\int_1^\infty \frac{1}{x(1+\ln x)^2}\,dx = \lim_{N\to\infty}\int_1^N \frac{1}{x(1+\ln x)^2}\,dx = \lim_{N\to\infty} -\frac{1}{1+\ln x}\Bigg|_1^N \quad \tfrac{1}{2}$$

$\frac{1}{2}$

$$= \lim_{N\to\infty}\left(-\frac{1}{1+\ln N}+1\right) \quad \tfrac{1}{2}$$

$$= 1 \quad \tfrac{1}{2}$$

$\int \frac{1}{x(1+\ln x)^2}\,dx$ $\quad u = 1+\ln x$ $\frac{1}{2}$, $du = \frac{1}{x}dx$

$= \int \frac{1}{u^2}\,du$ $\frac{1}{2}$

$= -\frac{1}{u} = -\frac{1}{1+\ln x}$

SO, $\int_1^\infty \frac{1}{x(1+\ln x)^2}\,dx$ CONV $\frac{1}{2}$

SO $\boxed{\sum_{n=1}^{\infty} \frac{1}{n(1+\ln n)^2} \text{ CONV (INTEGRAL TEST)}}$ $\frac{1}{2}$

SO $\boxed{\sum_{n=1}^{\infty} \left| \frac{\sin n^2}{n(1+\ln n)^2} \right| \text{ CONV (COMPARISON TEST)}}$ $\frac{1}{2}$

SO $\sum_{n=1}^{\infty} \frac{\sin n^2}{n(1+\ln n)^2}$ $\boxed{\text{CONV (ABSOLUTE CONVERGENCE TEST)}}$ $\frac{1}{2}$

☆ ALTERNATE PROOF THAT $\frac{1}{x(1+\ln x)^2}$ DECREASING

IF $1 \le x_1 < x_2$, THEN $0 \le \ln x_1 < \ln x_2$

SO $0 < (1+\ln x_1)^2 < (1+\ln x_2)^2$

SO $0 < x_1(1+\ln x_1)^2 < x_1(1+\ln x_2)^2 < x_2(1+\ln x_2)^2$

SO $\frac{1}{x_1(1+\ln x_1)^2} > \frac{1}{x_2(1+\ln x_2)^2}$

SO $\frac{1}{x(1+\ln x)^2}$ DECREASING

TALK TO ME IF YOU USED THIS METHOD ("HANDWAVING" NOT ACCEPTABLE)

[2][c] $\lim_{n\to\infty} \sqrt[n]{\left|\left(1-\sin\frac{2}{n}\right)^{n^3}\right|}$

$= \underline{\lim_{n\to\infty} \left(1-\sin\frac{2}{n}\right)^{n^2}}$ $\frac{1}{2}$

$= \lim_{n\to\infty} e^{\ln\left(1-\sin\frac{2}{n}\right)^{n^2}}$

$= \lim_{t\to-\infty} e^t$

1 $\underline{=0}\ \underline{<1}$ $\frac{1}{2}$

$\sum_{n=1}^{\infty}\left(1-\sin\frac{2}{n}\right)^{n^3}$ $\underline{\text{CONV (ROOT TEST)}}$ $\frac{1}{2}$

$\lim_{n\to\infty} n^2 \ln\left(1-\sin\frac{2}{n}\right)$

$\frac{1}{2}$ $= \underline{\lim_{n\to\infty} \frac{\ln\left(1-\sin\frac{2}{n}\right)}{n^{-2}}}$

$= \lim_{n\to\infty} \frac{\frac{1}{1-\sin\frac{2}{n}}\left(-\cos\frac{2}{n}\right)\left(-\frac{2}{n^2}\right)}{-\frac{2}{n^3}}$

$\frac{1}{2}$ $= \underline{\lim_{n\to\infty} -\frac{n\cos\frac{2}{n}}{1-\sin\frac{2}{n}}}\ \underline{=-\infty}$ $\frac{1}{2}$

$\cos\frac{2}{n}\to\cos 0\to 1$

$1-\sin\frac{2}{n}\to 1-\sin 0\to 1$